5. Other Cryptographic Constructions Relying on Coding Theory

- Code-Based Digital Signatures
- The Courtois-Finiasz-Sendrier (CFS) Construction
- Attacks against the CFS Scheme
- Parallel-CFS
- Stern’s Zero-Knowledge Identification Scheme
- An Efficient Provably Secure One-Way Function
- The Fast Syndrome-Based (FSB) Hash Function
Requirements for a Cryptographic Hash Function

A cryptographic hash function has the following properties:

• its input can be of arbitrary size
• its output is a hash of fixed size
• from a security point of view, it should be hard to:
  • find an input with a given hash (preimage attack)
  • find an input with the same hash as a given input (second preimage)
  • find two inputs with the same hash (collision attack)

In addition, it should, as much as possible:

• be fast in both software and hardware implementations
• be fast for both small and large inputs
• have a compact description
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Building a Cryptographic Hash Function

Building a function with arbitrary input length is tricky
→ usually, iterate a function with fixed input size on blocks of the input

The Merkle-Damgård Construction

One of the first hash function constructions:
- $f$ is a compression function
- easy to understand, simple security proofs
Building a Cryptographic Hash Function

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The Davies-Meyer Construction

Ideal for compact implementations:

- $E$ is a block cipher
- can reuse the same hardware
Building a Cryptographic Hash Function

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The Sponge Construction

- Maximum versatility
Overview of the Fast Syndrome-Based Hash Function

Uses the Merkle-Damgård construction.

Allows simple security analysis:
- properties of the compression function are transferred to the hash function
  - preimage resistance
  - second preimage resistance
  - collision resistance
  - analyse only the compression function
- has some drawbacks, but not so problematic:
  - long message collisions, multi-collisions...
Overview of the Fast Syndrome-Based Hash Function

Uses the Merkle-Damgård construction.

Uses the one-way function (previous session) with compression parameters.
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Uses the Merkle-Damgård construction.

Uses the one-way function (previous session) with compression parameters.

Adds a final compression function.
Description of FSB\textsubscript{256}

Compression function:

- the matrix $H$ is of size $r = 1024$ by $n = 2^{21}$
- the input of $s = 1792$ bits is encoded into a regular word of weight $w = 128$
  $\Rightarrow$ each position is coded on $\frac{s}{w} = \log \frac{n}{w} = 14$ bits
- the output of $r = 1024$ bits is the XOR of 128 columns of $H$
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**Chaining:**
- the message to hash is split in blocks of $s - r = 768$ bits
- a padding is added to get an integer number of blocks
  - includes the message length
- the IV is all 0
- the compression function is iterated on the blocks
- the final output of $r = 1024$ bits is input to Whirlpool
  - the final hash has 256 bits
Security of the Compression Function

Against (second) preimage:
- solve a regular instance of SD with weight 128 and a $1024 \times 2^{21}$ matrix
- best attack: GBA with complexity $2^{261} > 2^{256}$

Against collision:
- solve a regular instance of SD with weight 256 and a $1024 \times 2^{21}$ matrix
- best attack: ISD with complexity $2^{153} > 2^{128}$
Need for a Final Compression Function

A hash function is expected to have the following properties:

- a security of $2^{\frac{r}{2}}$ against collisions for an output of $r$ bits
  → this is the cost of a generic attack using a birthday algorithm
- it should be possible to truncate the output without losing security
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This is not the case for the compression function of FSB:

- if $w$ allows compression, GBA with 4 lists is always possible on weight $2w$
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Simply add a final compression:

- must be non-linear
- does not have to be collision/preimage resistant
Efficiency

Hashing speed:

- each 14 bits of input add a 1024 bit XOR
  - theoretically, could be as low as 10 cycles per byte (a 64 bit XOR per cycle)
- in practice, requires 300 cycles per input byte
  - around 10 MB/s hashing
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Size of the description:
- $H$ has a size of $2^{10} \times 2^{21}$ bits, that is 256 MB
  - this is way too much!
- instead a quasi-cyclic matrix is used
  - each $1024 \times 1024$ block is circulant
- only the first line of the matrix is needed
  - the description is 1024 times smaller: 256 kB
5. Other Cryptographic Constructions Relying on Coding Theory

We have seen several constructions relying on the hardness of Syndrome Decoding:
- McEliece, Niederreiter
- the CFS signature
- Stern’s identification scheme
- the FSB hash function

Many other applications of coding theory in cryptography:
- secret sharing
- linear diffusion in block ciphers
- fingerprinting and traitor tracing
- private information retrieval